

# Welcome to AP Calculus!

This packet is meant to help you review some of the mathematics that lead up to calculus. Of course, all mathematics up until this point has simply been a build-up to calculus, but these are the most important topics. Since the focus of this class must be the actual content of the AP test, this packet is meant only to be a review and not an in-depth course of study. This is the point in your math careers when telling a teacher, "But wait --- that was last year! You expect us to remember that?" is not going to cut it. The answer is, "**YES!!**" If you find yourself weak in any of these areas, **or feel that you were never taught this material**, then make sure to review/learn them and strengthen your understanding before August. Here are a couple of resources to help you out:

Paul's Online Notes has many helpful resources (this web site has great notes and problems that can help throughout the year, so I would keep it bookmarked):

<http://tutorial.math.lamar.edu/Extras/AlgebraTrigReview/AlgebraTrigIntro.aspx>

<http://tutorial.math.lamar.edu/Classes/Alg/Alg.aspx>

Math Mentor (more of a review web site):

<http://www.mastermathmentor.com/Calc/RUReady.ashx>

Topics that former Calculus students have really struggled with in the past, but that you need to know **very well** for AP Calculus:

- \***Factoring** (binomials, trinomials, trig expressions, etc.)
- \***Logarithmic rules** ( $\ln$  vs  $\log$ , basic rules to make things easier, etc.)
- \***Area and Volume** formulas for basic shapes
- \***Trig Identities** (yes you are expected to know these)
- \***Basic shapes** of graphs (parabolas, sine function, square root function, exponential, etc.)
- \***Solving inequalities** (not just linear)
- \***Exponent rules** (writing radicals as exponents, simplifying, etc.)
- \***Rational Functions** (Domain, range, asymptotes, holes, simplifying, etc.)

### **Frequently Asked Questions:**

1) ***Do I really need to know the stuff in this packet?***

Yes, you do. There is a **TEST** over this material on the very **first Tuesday** we come back to school!!

2) ***What topics are on the AP Calculus exam?***

The list of topics changes occasionally, but the College Board website ([www.collegeboard.org/ap/calculus](http://www.collegeboard.org/ap/calculus)) always has the current course description.

3) ***What's the difference between Calculus AB and BC?***

The Calculus BC curriculum contains considerably more material than the AB curriculum. This means that BC moves at a *very* rapid pace. Completing Calculus BC is equivalent to completing college Calculus I and (most of) Calculus II, whereas AB covers all of college Calculus I and a portion of Calculus II.

4) ***What kind of grade do I need to get on the AP exam in order to get college credit?***

Research, research, research!! Not all colleges/universities accept the same scores or give the same amount of credit for passing the exam. You have to check with your future school of choice in order to know what score is accepted and how much credit you will receive.

Students taking the BC exam will receive two scores: an overall BC score and an AB-subscore which gives a grade only on the AB material. It is possible for a student not to pass the entire exam, yet receive a passing AB score for which they could still get some college credit.

Have an enjoyable and restful summer. But also do some math! We look forward to teaching you next year.

The Grayson High School AP Calculus Team

**You are expected to be able to do all of these questions WITHOUT a calculator (except section XXI) Using a graphing calculator**

I) *Simplify the following fractions:*

1)  $\frac{1}{x} + \frac{1}{y} =$

2)  $\frac{1}{x} + \frac{1}{x^2} =$

3)  $\frac{\frac{1}{x} + 1}{\frac{1}{x}} =$

4)  $\frac{\frac{x}{x+y}}{x} =$

5)  $\frac{\frac{1}{x+h} + \frac{1}{x}}{x} =$

II) *Factor each expression:*

6)  $x^2 - 16$

7)  $x^2 - x - 6$

8)  $6x^2 - x - 2$

9)  $4x^3 - 19x^2 - 5x$

10)  $x^2 + 9$

11)  $x^4 - 13x^2 - 30$

12)  $x^3 + 27$

13)  $x^3 - 8$

14)  $(2x-3)^3(x+1) + (x-3)(2x-3)^2$

15)  $(3x-2)^{-4}(x+3) + (x+3)^2(3x-2)^{-3}$

**III) Solve the following equations/inequalities for  $x$ :**

**16)**  $x^2 + 5x - 24 = 0$

**17)**  $x^2 - 9 = 5$

**18)**  $3x^2 - 5x - 2 = 0$

**19)**  $x^2 - 4x = 0$

**20)**  $(x-1)(x^2 - 11x + 30) = 0$

**21)**  $\sqrt{x+1} = 41$

**22)**  $\frac{y}{x+1} = \frac{z}{x}$

**23)**  $\sqrt[3]{x+1} - 4 = -1$

**24)**  $x^{-2} = \frac{1}{9}$

**25)**  $2\sqrt{x} = x - 3$

**26)**  $\frac{8+x}{x} - 5 = 0$

**27)**  $x^{-1} = -3$

**28)**  $x^{\frac{4}{3}} = 81$

**29)**  $3x^2 - 6x - 24 \leq 0$

**30)**  $\frac{2x-1}{(x+2)(x^2+3)} = 0$

**31)**  $x^3 - 2x^2 - 5x + 10 = 0$

IV) *Are the following expressions equal to  $\ln 4$  ?*

32)  $2 \ln 2$

33)  $\frac{\ln 8}{\ln 2}$

34)  $\ln 8 - \ln 2$

35)  $\ln 4 + \ln 1$

36)  $\ln 4 \cdot \ln 1$

37)  $(\ln 2)^2$

V) *Write an equation of a line based on the given information:*

38) Find the equation of the line that has a slope of 5 and passes through the point (3, -4).

39) Find the equation of the line that passes through the points (4, 1) and (3, -2).

40) Find the equation of the line that passes through the points (-2, 1) and is parallel to the line  $4x + 2y = -1$ .

41) Find the equation of the line that has a slope of 0 and passes through the point (-5, 1).

42) Find the equation of the line that passes through the origin and is perpendicular to the line  $3x + 4y = -7$ .

43) Find the equation of the line that has an undefined slope and passes through the point (4, -5).

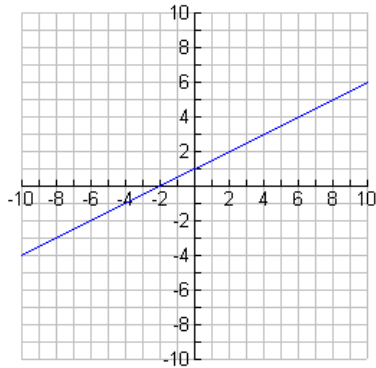
**VI) Intercepts:**

**44) Find the  $x$  and  $y$  intercepts of  $x^2 + y^2 = 9$ .**

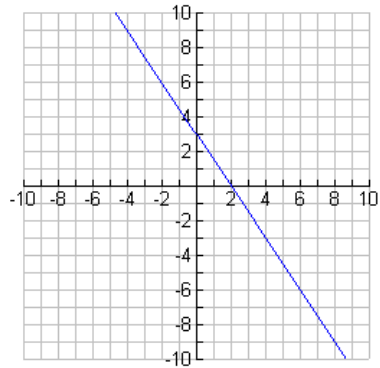
**45) Find the equation of the line that has an  $x$ -intercept of 5 and a  $y$ -intercept of 3.**

**VII) Write the equation for the following graphs:**

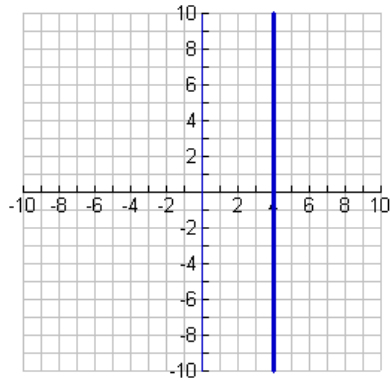
**46)**



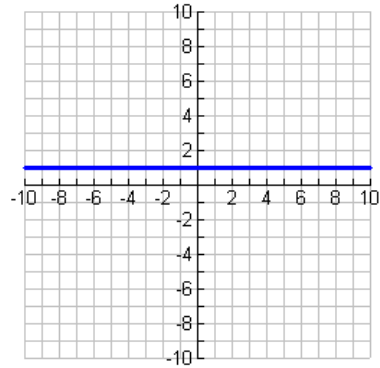
**47)**



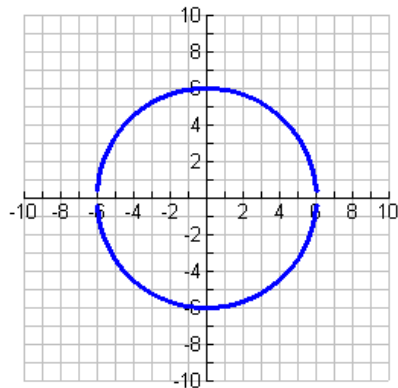
**48)**



**49)**



**50)**



**VIII) Given the slope, sketch the following lines:**

**51) Sketch a line with a slope of 2.**

**52) Sketch a line with a slope of  $\frac{1}{2}$ .**

**53) Sketch a line with a slope of -2.**

**54) Sketch a line with a slope of  $-\frac{1}{2}$ .**

**IX) Sketch the following graphs:**

**55)  $f(x) = 3x + 1$**

**56)  $f(x) = x^2$**

**57)  $f(x) = |x|$**

**58)  $f(x) = x^3$**

**59)  $x = 3$**

**60)  $y = -4$**

**61)  $f(x) = \ln x$**

**62)  $f(x) = \sqrt{x}$**

**63)  $f(x) = \frac{1}{x}$**

**64)  $f(x) = \frac{1}{x^2}$**

**65)  $f(x) = |x + 1|$**

**66)  $f(x) = x^2 + 2x - 3$**

**67)  $f(x) = x^3 + 1$**

**68)  $f(x) = (x + 1)^2$**

**69)  $f(x) = -x^2 + 1$**

**70)  $f(x) = (x + 1)^{1/3}$**

**71)  $f(x) = x^{2/3}$**

**72)  $f(x) = e^x$**

**73)  $f(x) = -\frac{1}{x}$**

**74)  $x^2 + y^2 = 25$**

**X) Rewrite the following functions without absolute values:**

**75)  $f(x) = |x|$**

**76)  $f(x) = |x - 1|$**

**XI) Find the domain and range of each function:**

77)  $f(x) = x - 1$

78)  $f(x) = \frac{1}{x}$

79)  $f(x) = \frac{1}{x^2 + 1}$

80)  $f(x) = e^x$

81)  $f(x) = \sqrt{x - 4}$

82)  $f(x) = |x - 1| + 2$

83)  $f(x) = \ln x$

84)  $f(x) = \sqrt{x^2 - 3x - 4}$

85)  $f(x) = \frac{1}{x + 6} - 10$

**XII) Find the inverse of each function:**

86)  $f(x) = x + 3$

87)  $f(x) = \sqrt{x}$

88)  $f(x) = \frac{x}{x + 2}$

89)  $f(x) = \ln x$

**XIII) Find the compositions of the function if:**

$$f(x) = x^3 + 1, \quad g(x) = x^2 - 2, \quad \text{and} \quad j(x) = x + 3$$

90)  $f(2)$

91)  $f(j(x))$

92)  $f(j(2))$

93)  $g(g(x))$

94)  $f(x + h)$

95)  $\frac{f(x + h) - f(x)}{h}$

**XIV) Solve the simultaneous equations:**



**96a)**  $2x + 3y = 8$   
 $x + 2y = 5$

**96b)**  $y = x^2 + 2x + 9$   
 $7x + y = 19$

**97a)** The length,  $l$ , of a certain rectangle is twice the width,  $w$ . Write an equation for the perimeter the rectangle as a function of the width,  $w$ .

**97b)** If the area of the rectangle described above is 50 square feet, find the length and the width of the rectangle.

**XV) Intersection of curves:**

**98) Find the point of intersection between the lines  $y = x + 1$  and  $3y - x = 5$ .**

**99) Find the point of intersection between the lines  $y = x + 7$  and the curve  $y = x^2 + 2x + 5$ . Also sketch the area between the graphs.**

**XVI) What do the following mean?**

- 100) a graph is in the first quadrant**
- 101)  $f(2) = 5$**
- 102) an expression is a function**
- 103) a zero of a function is 4**
- 104)  $y$  is directly proportional to  $x$  (give an example)**
- 105) the coefficient of the third term is 5 (give an example)**
- 106) a function has only one root**
- 107) a function is a polynomial**
- 108) two triangles are similar**
- 109) a function is even**
- 110) a function is odd**

**XVII) What are the following formulas?**

- 111) Quadratic formula
- 112) Pythagorean Theorem
- 113) the hypotenuse of a 45-45-90 isosceles right triangle with a leg of length  $x$ .
- 114) the hypotenuse of a 30-60-90 right triangle with shortest leg having a length of  $x$ .
- 115) the volume of a sphere
- 116) the volume of a cylinder
- 117) the volume of a cone
- 118) the volume of a box with a square base
- 119) the surface area of a sphere
- 120) the surface area of a cylinder with no top
- 121) the area of a triangle
- 122) the area of a trapezoid
- 123) the cross section through the center of a sphere
- 124) the volume of a prism that has an equilateral triangle with side of length  $x$  and height of length  $y$
- 125) area of an equilateral triangle in terms of the length of a side  $s$

**XVIII) Solve using similar triangles:**

126) A six foot man is standing 10 feet away from a 20 foot lamppost. What is the length of his shadow?

127) Water is dripping out of a conical figure that has a diameter of 8 inches and a height of 12 inches. When the depth of the water is 8 inches, what is the radius of the water?

**XIX) Find the equations of the horizontal and vertical asymptotes of each function:**

128)  $y = \frac{1}{x-1}$

129)  $y = \frac{x^3}{x^3-1}$

**XX) Exponent Rules: Which of the following are true?**

130)  $x^0 = 1$

131)  $x^{-2} = \frac{1}{x^2}$

132)  $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$

133)  $x^5 \cdot x^3 = x^{15}$

134)  $x^5 \cdot y^5 = (xy)^5$

135)  $(x^3)^5 = x^8$

136)  $x^{5-w} = \frac{x^5}{x^w}$

137)  $x^{t+5} = (x^t)^5$

138)  $\sqrt{\frac{9}{4}} = \frac{3}{2}$

139)  $(4x)^{\frac{1}{2}} = 2x$

140)  $\sqrt{\frac{1}{x}} = x^{-\frac{1}{2}}$

141)  $\sqrt{x^2} = x$

142)  $\sqrt{x^2 - 25} = x - 5$

143)  $x^{\frac{4}{3}} = \sqrt[4]{x^3}$

144)  $\left(x^{\frac{1}{2}} + y^{\frac{1}{2}}\right)^2 = x + y$

145)  $x^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{x^2}}$

146)  $e^{\ln x^2} = x^2$

147)  $\ln e^3 = 3$

148)  $e^{2\ln 2 - \ln 5} = \frac{4}{5}$

149)  $\ln x^2 = (\ln x)^2$

150) Expand using the properties of logarithms:  $\ln \sqrt[3]{\frac{(3x+7)^4 (x+10)^3}{(5x-8)^2}}$

151) Condense into a single logarithmic expression using the properties of logarithms:  
 $17 \ln x - \frac{2}{3} \ln(x^5 + 5)$

**XXI) Using the graphing calculator:**

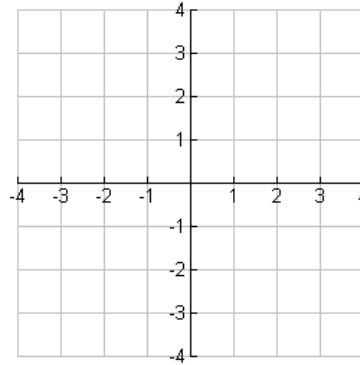
**152)** Graph  $y = 0.1x^3 + 2x^2 - x - 3$  on the  $x$ - $y$  plane on the right:

**153)** Find the roots of the equation above.

**154)** Find the point of intersection for the graphs  $y = x^3 + x - 3$  and  $y = 2x + 4$ .

**155)** Find the maximum value for the graph  $f(x) = -x^4 + x - 4$ .

**156)** For the function in #155, find the intervals on which  $f(x)$  is increasing.



**XXII) What are the following trigonometric identities?**

**157)**  $\sec x =$

**158)**  $\csc x =$

**159)**  $\tan x =$

**160)**  $\cot x =$

**161)**  $\cos^2 x - 1 =$

**162)**  $\sec^2 x - 1 =$

**163)**  $\cot^2 x + 1 =$

**XXIII) Evaluate the following expressions:**

**164)**  $\sin\left(\frac{\pi}{6}\right)$

**165)**  $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

**166)**  $\tan\left(\frac{7\pi}{6}\right)$

**167)**  $\cos(0)$

**168)**  $\cos\left(\frac{\pi}{4}\right)$

**169)**  $\csc\left(\frac{-5\pi}{6}\right)$

**170)**  $\sec(\pi)$

**171)**  $\cot\left(\frac{-\pi}{2}\right)$

**172)**  $\sin^{-1}\left(\frac{1}{2}\right)$

**173)**  $\tan\left(\frac{\pi}{2}\right)$

**174)**  $\sin^2\left(\frac{5\pi}{6}\right)$

**175)**  $\cot\left(\frac{2\pi}{3}\right)$

**176)**  $\sin\left(\frac{\pi}{2}\right)$

**177)**  $\cot^{-1}(-1)$

**178)**  $\sec\left(\frac{3\pi}{4}\right)$

**179)**  $\tan^{-1}(-1)$

**180)**  $\csc(\pi)$

**181)**  $\sec^2\left(\frac{\pi}{4}\right)$

**XXIV)** *Sketch one period of the following trigonometric graphs:*

**182)**  $y = \sin x$

**183)**  $y = \cos x$

**184)**  $y = \tan x$

**185)**  $y = \sec x$

**186)**  $y = \csc x$

**187)**  $y = \cot x$

**XXV)** *Solve the following trigonometric equations for the given domain:*

**188)**  $\sin x = \cos x$  on  $[0, 2\pi]$

**AP Calculus Summer Packet**  
**Answer Key**

**Reminders:**

1. This is **not** an assignment.
2. This will **not** be collected.
3. You **WILL** have a test over this material the first week we come back to school (and there will possibly be no “in-class time” to work/ask questions).
4. You are expected to be able to do all the questions in this packet **without** a calculator except for those in Section XXI.

**I) Simplifying fractions:**

1)  $\frac{y+x}{xy}$     2)  $\frac{x+1}{x^2}$     3)  $1+x$     4)  $\frac{1}{x+y}$     5)  $\frac{2x+h}{x^2(x+h)}$

**II) Factoring:**

6)  $(x+4)(x-4)$     7)  $(x-3)(x+2)$     8)  $(2x+1)(3x-2)$     9)  $x(x-5)(4x+1)$

10) Does not factor    11)  $(x^2-15)(x^2+2)$     12)  $(x+3)(x^2-3x+9)$

13)  $(x-2)(x^2+2x+4)$     14)  $2(x^2-3)(2x-3)^2$     15)  $\frac{(x+3)(3x^2+7x-5)}{(3x-2)^4}$

**III) Solving:**

16)  $x = -8, 3$     17)  $x = \pm\sqrt{14}$     18)  $x = -\frac{1}{3}, 2$     19)  $x = 0, 4$

20)  $x = 1, 5, 6$     21)  $x = 1600$     22)  $x = \frac{z}{y-z}$     23)  $x = 26$

24)  $x = \pm 3$     25)  $x = 9$  ( $x = 1$  is extraneous.)    26)  $x = 2$

27)  $x = -\frac{1}{3}$     28)  $x = 27$     29)  $[-2, 4]$     30)  $x = \frac{1}{2}$     31)  $x = 2, \pm\sqrt{5}$

**IV) Are the following expressions equal to  $\ln 4$ ?**

32) **yes**    33) **no**    34) **yes**    35) **yes**    36) **no**    37) **no**

**V) Equations of lines:**

38)  $y + 4 = 5(x - 3)$  or  $y = 5x - 19$       39)  $y - 1 = 3(x - 4)$  or  $y + 2 = 3(x - 3)$  or  $y = 3x - 11$

40)  $y - 1 = -2(x + 2)$  or  $y = -2x - 3$       41)  $y = 1$       42)  $y = \frac{4}{3}x$       43)  $x = 4$

**VI) Intercepts:**

44)  $x$ -intercepts:  $(3, 0)$  and  $(-3, 0)$ .  $y$ -intercepts:  $(0, 3)$  and  $(0, -3)$

45)  $y - 3 = -\frac{3}{5}(x - 0)$  or  $y - 0 = -\frac{3}{5}(x - 5)$  or  $y = -\frac{3}{5}x + 3$

**VII) Equations of graphs:**

46)  $y = \frac{1}{2}x + 1$       47)  $y = -\frac{3}{2}x + 3$       48)  $x = 4$       49)  $y = 1$       50)  $x^2 + y^2 = 36$

**VIII) Given the slope, sketch the following lines:**

51) – 54) [If you do not trust your sketches, see your instructor.]

**IX) Sketch the following graphs:**

55) – 74) [You can check your answers with a graphing calculator, but please know that you are expected to know what these look like without the use of a graphing calculator.]

**X) Absolute value:**

75)  $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

76)  $|x - 1| = \begin{cases} x - 1, & x \geq 1 \\ -x + 1, & x < 1 \end{cases}$

**XI) Domain and range:**

77)  $D: \mathbb{R}, R: \mathbb{R}$       78)  $D: \mathbb{R} \neq 0, R: \mathbb{R} \neq 0$       79)  $D: \mathbb{R}, R: 0 < y \leq 1$       80)  $D: \mathbb{R}, R: \mathbb{R} > 0$   
81)  $D: \mathbb{R} \geq 4, R: \mathbb{R} \geq 0$       82)  $D: \mathbb{R}, R: \mathbb{R} \geq 2$       83)  $D: \mathbb{R} > 0, R: \mathbb{R}$   
84)  $D: (-\infty, -1], [4, \infty), R: \mathbb{R} \geq 0$       85)  $D: \mathbb{R} \neq -6, R: \mathbb{R} \neq -10$

**XII) Inverses:**

86)  $f^{-1}(x) = x - 3$       87)  $f^{-1}(x) = x^2$       88)  $f^{-1}(x) = \frac{-2x}{x-1}$       89)  $f^{-1}(x) = e^x$

**XIII) Compositions of functions if:**

$f(x) = x^3 + 1$  ,  $g(x) = x^2 - 2$  , and  $j(x) = x + 3$

90)  $f(2) = 9$       91)  $f(j(x)) = (x+3)^3 + 1$       92)  $f(j(2)) = 126$

93)  $g(g(x)) = x^4 - 4x^2 + 2$       94)  $f(x+h) = x^3 + 3x^2h + 3xh^2 + h^3 + 1$

95)  $\frac{f(x+h) - f(x)}{h} = 3x^2 + 3xh + h^2$

**XIV) Simultaneous equations:**

96a)  $(1, 2)$       96b)  $(-10, 89)$  and  $(1, 12)$

97a)  $P = 6w$       97b)  $w = 5$  feet and  $l = 10$  feet

**XV) Intersection of curves:**

98)  $(1, 2)$       99)  $(-2, 5)$  and  $(1, 8)$

**XVI) What do the following mean?**

100) – 110) [If you do not trust your answers, see your instructor.]

**XVII) What are the following formulas?**

111)  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$       112)  $a^2 + b^2 = c^2$       113)  $x\sqrt{2}$       114)  $2x$

115)  $V = \frac{4}{3}\pi r^3$       116)  $V = \pi r^2 h$       117)  $V = \frac{1}{3}\pi r^2 h$       118)  $V = x^2 h$

119)  $SA = 4\pi r^2$       120)  $SA = \pi r^2 + 2\pi r h$       121)  $A = \frac{1}{2}bh$       122)  $A = \frac{1}{2}h(b_1 + b_2)$

123)  $A = \pi r^2$       124) [Ambiguous question--don't worry about this one.]      125)  $A = \frac{s^2\sqrt{3}}{4}$

**XVIII) Solve using similar triangles:**



126)  $30/7$  feet      127)  $8/3$  inches

**XIX) Find the equations of the horizontal and vertical asymptotes of each function:**

128) H.A.:  $y = 0$ , V.A.:  $x = 1$

129) H.A.:  $y = 1$ , V.A.:  $x = 1$

**XX) Exponent Rules: Which of the following are true?**

130) True    131) True    132) **NO !!!**    133) No    134) True    135) **No**

136) True    137) No    138) True    139) No    140) True

141) Technically, this is false.  $\sqrt{x^2} = |x|$  because we do not know if  $x$  is positive or negative. Just be careful.

142) **NO!!!**    143) No    144) **NO!!!**    145) True    146) True    147) True

148) True    149) No

150)  $\frac{1}{3} [4 \ln(3x+7) + 3 \ln(x+10) - 2 \ln(5x-8)]$     151)  $\ln \left( \frac{x^{17}}{\sqrt[3]{(x^5+5)^2}} \right)$

**XXI) Using the graphing calculator:**

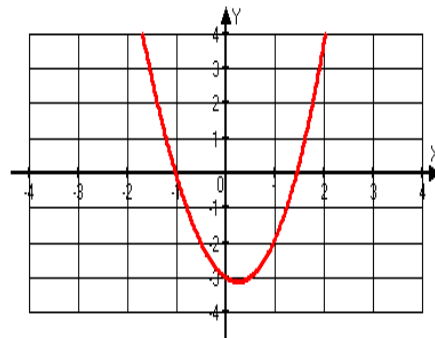
152) See graph.

153) -20.418 (not shown in the graph -- danger of graphing calculator), -1.021, 1.439

154) (2.087, 8.173)

155) -3.528

156)  $(-\infty, 0.630)$



**XXII) What are the following trigonometric identities?**

$$157) \sec x = \frac{1}{\cos x} \quad 158) \csc x = \frac{1}{\sin x} \quad 159) \tan x = \frac{\sin x}{\cos x} \quad 160) \cot x = \frac{\cos x}{\sin x}$$

$$161) \cos^2 x - 1 = -\sin^2 x \quad 162) \sec^2 x - 1 = \tan^2 x \quad 163) \cot^2 x + 1 = \csc^2 x$$

**XXIII) Evaluate the following expressions:**

$$164) \frac{1}{2} \quad 165) \frac{\pi}{6} \quad 166) \frac{\sqrt{3}}{3} \quad 167) 1 \quad 168) \frac{\sqrt{2}}{2} \quad 169) -2$$

$$170) -1 \quad 171) 0 \quad 172) \frac{\pi}{6} \quad 173) \text{undefined} \quad 174) \frac{1}{4}$$

$$175) -\frac{\sqrt{3}}{3} \quad 176) 1 \quad 177) \frac{3\pi}{4} \quad 178) -\sqrt{2} \quad 179) -\frac{\pi}{4}$$

$$180) \text{undefined} \quad 181) 2$$

**XXIV) Sketch one period of the following trigonometric graphs:**

**182) – 187)** [Use the internet resources or check with your instructor if you do not know how to graph these by hand.]

**XXV) Solve the following trigonometric equations for the given domain:**

$$188) x = \frac{\pi}{4}, \frac{5\pi}{4}$$